



IB MYP YEAR 5

YEAR 10 Extended  
Mathematics

Assessment #2  
**TRANSFORMATION OF FUNCTIONS**

Name:

Cindy Cheng

( 10 Place )

Teacher:

Ms. Li, Mr. So & Mr. Wong

Date of task:

Friday, November 2, 2012

Time allowed:

60 minutes

Student's Performance in Different Criterion

<b>B</b>	4	<b>C</b>	4
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**INSTRUCTIONS:**

- ◆ Read the instructions for all questions carefully.
- ◆ Show all work, steps and proper units.
- ◆ Ask the teacher for scrap paper, but any work on the scrap paper will **NOT** be marked.
- ◆ Write in **PENCIL**.
- ◆ **GDC** is allowed.
- ◆ Allowed to use **non-electronic dictionary**.

**ASSESSMENT:**

- ◆ Read the criteria descriptors carefully before you start your work. This will give you a clear understanding of what is required and what a quality piece of work for this task must include. This way you give yourself the best chance of achieving the highest level in this task.
- ◆ This task assesses Criteria **B & C** considering ALL the questions.
  - ✧ Criterion **C** will be assessed as an **overall impression** on the presentation of work in this assessment.

Criterion B: INVESTIGATING PATTERNS

Achievement level	Task Specific Rubric	IBO Published Descriptor	Student's self-evaluation
0	The student does not reach a standard described by any of the descriptors given below.	The student does not reach a standard described by any of the descriptors given below.	(0-8)
1-2 Do Maths	The student is able to <ul style="list-style-type: none"> <li>Find <b>some</b> of the equations of the curves in <b>part A</b>.</li> <li>Recognize and describe <b>some</b> of the transformations in <b>part A</b>.</li> </ul>	The student <b>applies, with some guidance</b> , mathematical problem-solving techniques to recognize <b>simple</b> patterns.	
3-4 General Rule	The student is able to <ul style="list-style-type: none"> <li>Do <b>most</b> of the questions in <b>Part A</b> and <b>some</b> questions in <b>Part B</b>.</li> <li>Apply knowledge to unfamiliar situations in <b>Q10</b> and <b>Q11</b>.</li> </ul>	The student <ul style="list-style-type: none"> <li><b>selects and applies</b> mathematical problem-solving techniques to recognize patterns, and</li> <li><b>suggests</b> relationships or general rules.</li> </ul>	Teacher's Final Grade
5-6 Test it	The student is able to <ul style="list-style-type: none"> <li>Fulfill the requirements above.</li> <li>Deduce a general form for <b>Q12</b>.</li> <li>Apply and justify the general form in <b>Q13</b>.</li> <li>Find another transformation that works in <b>Q14</b>.</li> </ul>	The student <ul style="list-style-type: none"> <li><b>selects and applies</b> mathematical problem-solving techniques to recognize patterns,</li> <li><b>describes</b> them as relationships or general rules, and</li> <li><b>draws conclusions</b> consistent with findings.</li> </ul>	(0-8)
7-8 Prove it	The student is able to <ul style="list-style-type: none"> <li>Fulfill the requirements above</li> <li>Prove mathematically the answer in <b>Q15</b>.</li> <li>Identify the unfamiliar transformation in <b>Q16</b> and find its equation in <b>Q17</b>.</li> </ul>	The student <ul style="list-style-type: none"> <li><b>selects and applies</b> mathematical problem-solving techniques to recognize patterns,</li> <li><b>describes</b> them as relationships or general rules,</li> <li><b>draws conclusions</b> consistent with findings, and</li> <li><b>provides justifications or proofs</b>.</li> </ul>	

Criterion C: COMMUNICATION IN MATHEMATICS

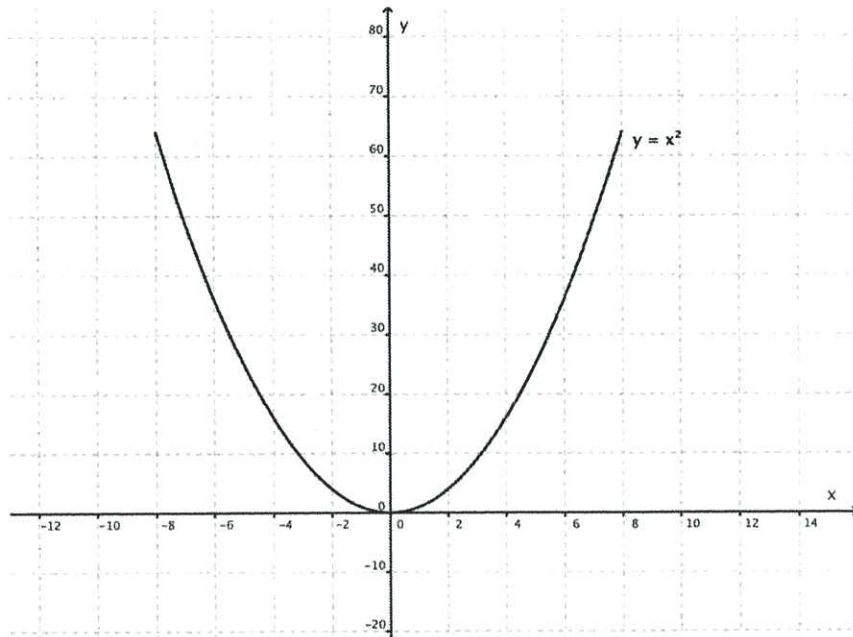
Achievement level	Task Specific Rubric	IBO Published Descriptor	Student's self-evaluation
0	The student does not reach a standard described by any of the descriptors given below.	The student does not reach a standard described by any of the descriptors given below.	
1–2	There are <b>some errors</b> or inconsistencies in use of <b>terminology</b> . There are <b>some errors</b> in the writing of <b>equations</b> . Narrative is <b>difficult to follow</b> .	<ul style="list-style-type: none"> <li>The student shows <b>basic</b> use of mathematical language <b>and/or</b> forms of mathematical representation.</li> <li>The lines of reasoning are <b>difficult to follow</b>.</li> </ul>	
3–4	Generally students use <b>correct terminology</b> accurately, with only a <b>few errors</b> . Equations are <b>mostly</b> written <b>clearly</b> and accurately. Narrative can be <b>followed</b> , and diagrams are <b>clear</b> and labeled.	<ul style="list-style-type: none"> <li>The student shows <b>sufficient</b> use of mathematical language <b>and</b> forms of mathematical representation.</li> <li>The lines of reasoning are <b>clear</b> though not always <b>logical</b> or <b>complete</b>.</li> <li>The student moves between different forms of representation <b>with some success</b>.</li> </ul>	Teacher's Final Grade
5–6	The student uses the correct terminology accurately for <b>most of the problems</b> . Equations are <b>clear</b> and <b>accurate</b> . Narratives are <b>concise, logical</b> and <b>complete</b> .  <b>All</b> diagrams are <b>clear</b> and labeled	<ul style="list-style-type: none"> <li>The student shows <b>good</b> use of mathematical language <b>and</b> forms of mathematical representation.</li> <li>The lines of reasoning are <b>concise, logical</b> and <b>complete</b>.</li> <li>The student moves <b>effectively</b> between different forms of representation.</li> </ul>	(0-6)



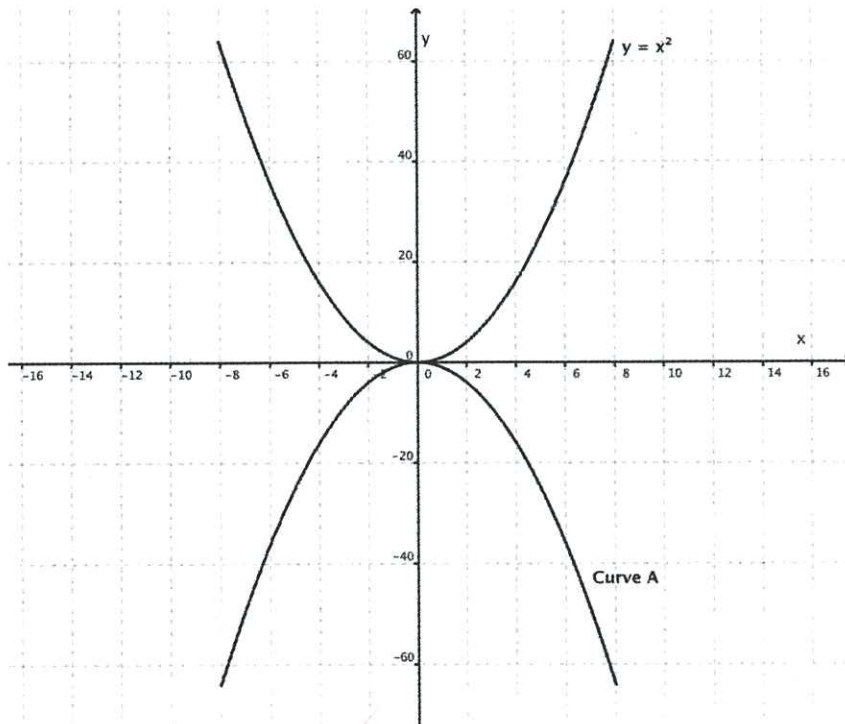
## PART A (Suggested time: 20 minutes)

When the designers of the Hong Kong MTR were creating the now-famous logo, they decided to use transformations of functions.

They started with a parabola as in the graph below:



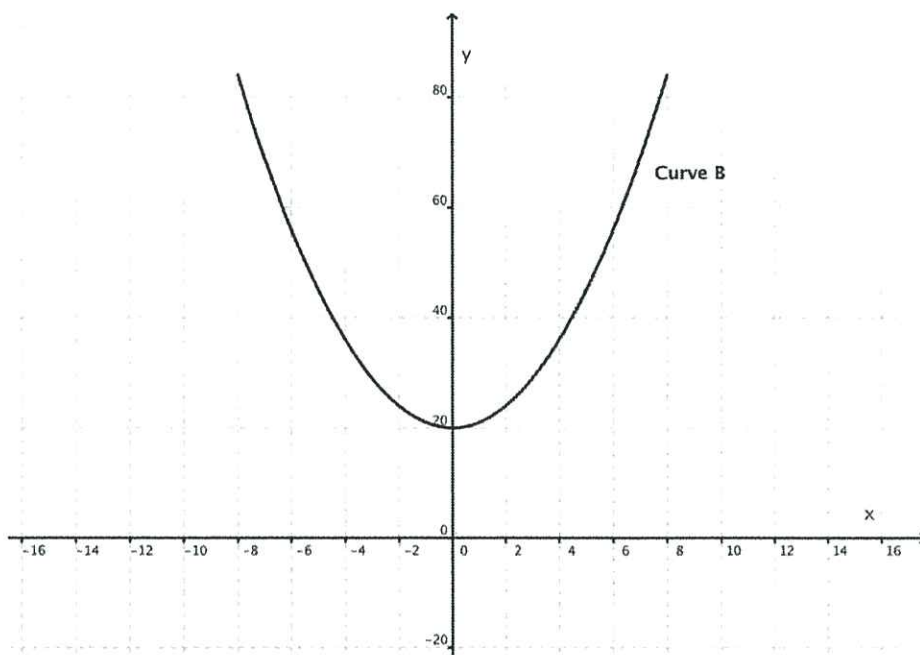
They then reflected this curve by the x-axis:



1. If the **starting curve** has the equation  $y = x^2$ , what is the **equation of curve A**?

$y = -x^2$

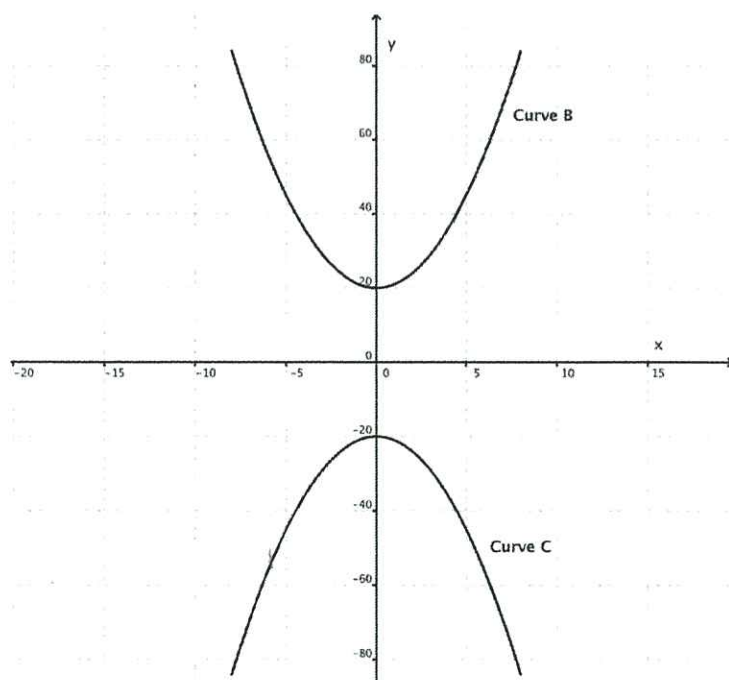
The designers then performed a transformation from their **starting curve**. It became this:



2. What do you think the **equation of curve B** is?

$$y = x^2 + 20$$

After the designers **reflected curve B** by the **x-axis**, they got **curve C**:

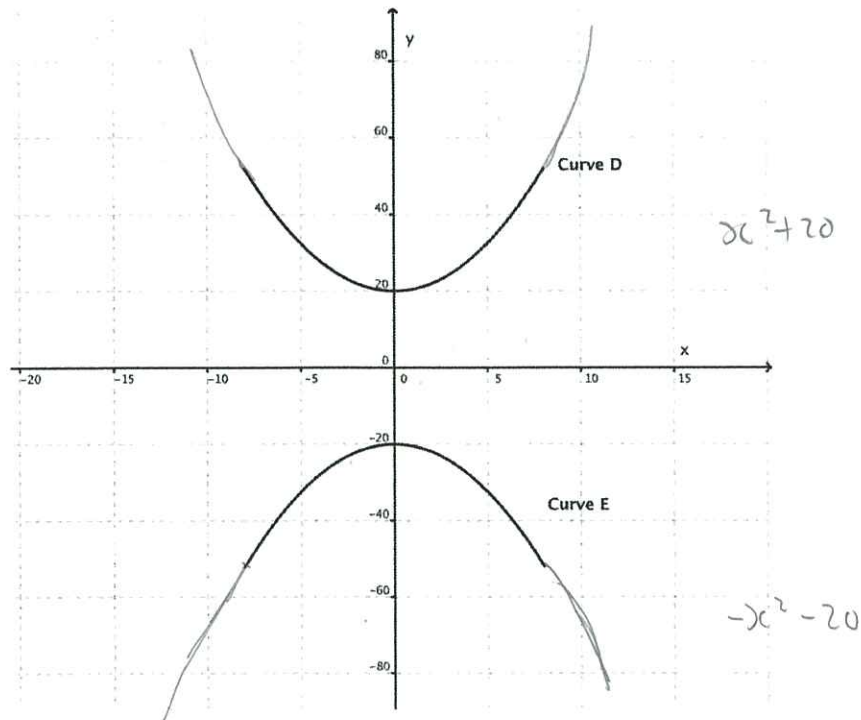


3. What do you think the **equation of curve C** is?

$$y = -(x^2 + 20)$$

$$y = -x^2 - 20$$

The designers then went on to make one further alteration. The diagram below shows the results of that alteration. (Curves D and E are sketched on the same axes as B and C were on the last graph):



4. Describe in words what this last alteration was (to make **curve B become curve D** and **C become E**)?

The designers enlarged curve B and C  $\frac{5}{4}$  times along the x-axis to create curve D and E. Then they cut short the opening of the curve B and C to create curve D and E.

about 2 units of

5. What do you think the **equation of curve D** is?

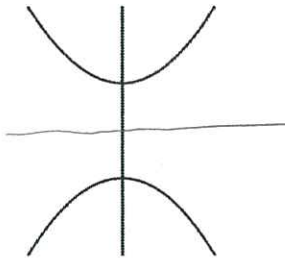
~~$y = \left(-\frac{5}{4}x\right)^2 + 20$~~

$y = \frac{x^2}{2} + 20$

6. What do you think the **equation of curve E** is?

~~$y = -\left(\frac{3}{4}x\right)^2 - 20$~~

$y = -\frac{x^2}{2} - 20$

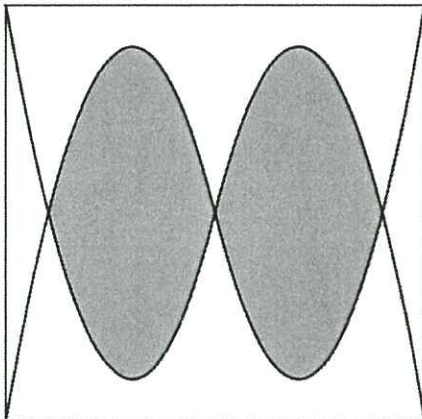


7. The "finished" logo is on the left. Describe fully the symmetry of the design

The "finished" logo is symmetric in 2 ways, firstly a horizontal line along the middle shows the reflection along the horizontal line. Secondly, the vertical line in the middle, shows the symmetric reflection of the two curves along the vertical line. Therefore the logo is symmetry.

## PART B (Suggested time: 40 minutes)

Here is a logo of a well-known sportswear company:

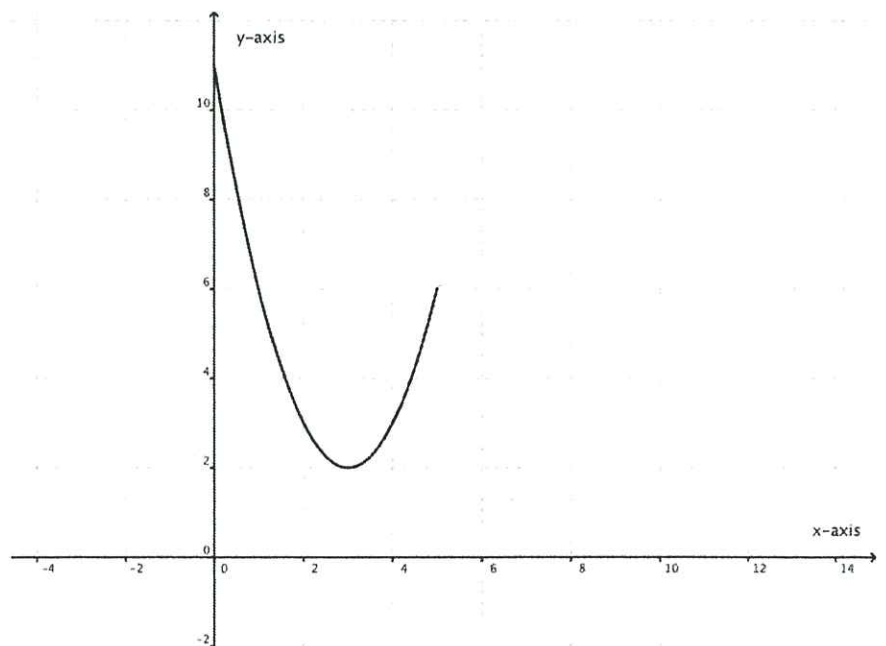


Two students, Ahmed and Delinda, are interested in the mathematics of the curves that make up the logo, and the relationship between the curves.

Ahmed believes that the design is really based on a simple curve (a quadratic), which is then transformed using simple transformations.

He says that the "basic unit" of the design below is

$$y = x^2 - 6x + 11 \quad 0 \leq x \leq 5$$



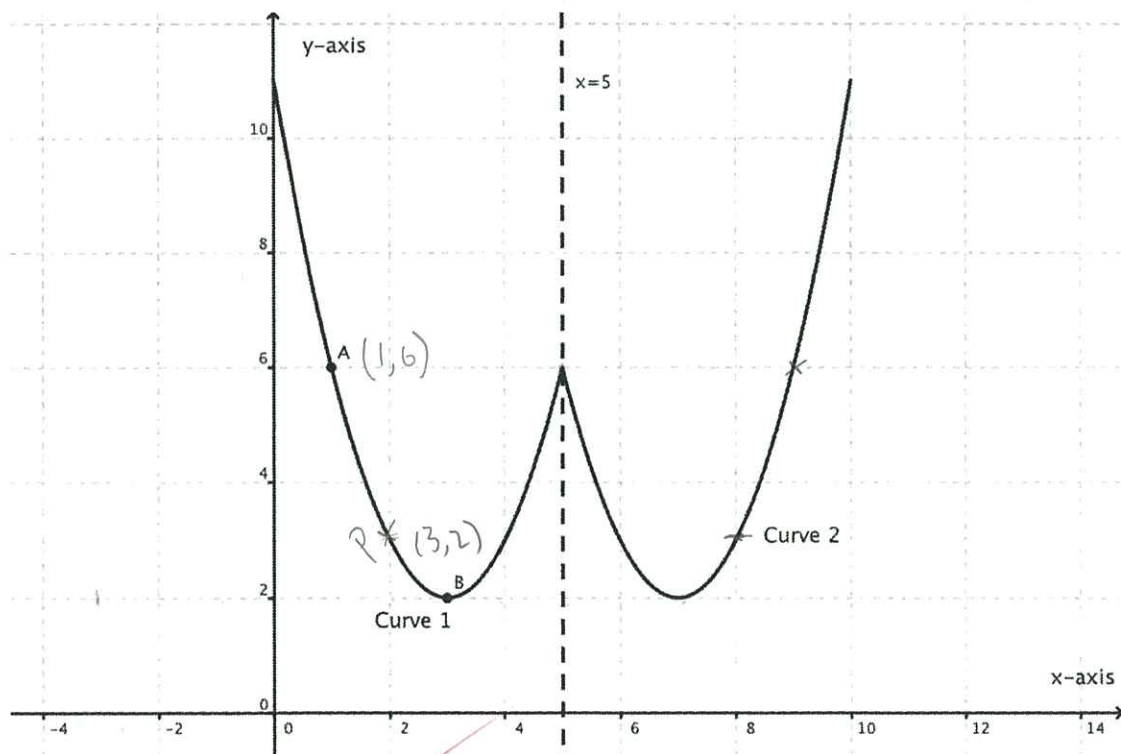
8. Show that the equation  $y = x^2 - 6x + 11$  can be written as the **second form**  $y = (x - 3)^2 + 2$ .

$$\begin{aligned}
 y &= x^2 - 6x + 11 \\
 h &= \frac{-(-6)}{2(1)} = 3 & k &= \frac{4(1)(11) - (-6)^2}{4(1)} = \frac{44 - 36}{4} = 2 \\
 \therefore y &= a(x - h)^2 + k \\
 \therefore y &= (x - 3)^2 + 2
 \end{aligned}$$

9. Describe how this second form of the equation matches up with key features of the graph?

This second form of the equation shows the vertex of the curve, which is  $(3, 2)$ . Also allowing the students to calculate the equation after any form of translation to create the full logo.

He goes on to say that this curve is **reflected in the line  $x = 5$**  as shown below:



10. A is the point  $(1, 6)$ . What are the coordinates of the image of A after reflection in  $x = 5$ ?

$(9, 6)$



11. B is the point (3, 2). What are the coordinates of the image of B after reflection in  $x = 5$ ?

(5, 2)

12. P is the general point (x, y). What are the coordinates of the image of P after reflection in  $x = 5$ ?

if  $p = (2, 3)$  coordinates of the image of  $p = (8, 3)$

13. Ahmed takes the x-coordinate of his answer to Q12, and substitutes this in for x in the original equation  $y = x^2 - 6x + 11$ . He (correctly) believes that it gives him the equation of curve 2. What equation for curve 2 does Ahmed get, and what range of values of x does it apply to?

$$y = 8^2 - 6(8) + 11$$

$$y = 27$$

$$x > 8$$

Delinda believes that Curve 1 in the diagram above can be transformed into Curve 2 **by a different way**.

14. What transformation(s) is/are Delinda thinking of? Give as many details as possible.

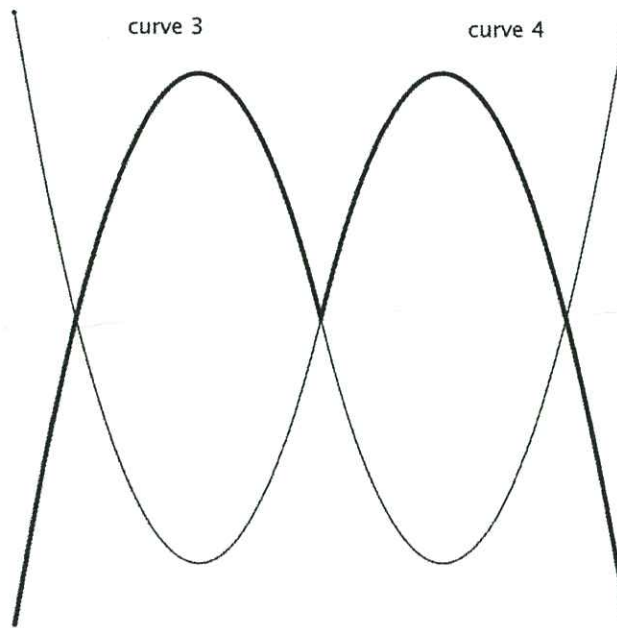
Delinda is thinking to first reflect curve 1 along y-axis, which gives her an image of curve 1 on the other side of y axis. The equation of the image of curve 1 will be  $y = (-x - 3)^2 + 2$ . Then translate the image of the curve 10 units to the right, creating the exact same curve 2 as Ahmed's.

of substituting

15. Is it possible for both Ahmed and Delinda to be right? Explain your answer.

I think it is possible for both Ahmed and Delinda to be right, because in Ahmed's way, he reflected curve 1 along the line of symmetry between curve 1 and curve 2, therefore it can create the curves of the logo. On the other hand Delinda's way is right too, because first reflecting curve 1 along y-axis, may create an image of curve 1, then to make it to the logo, she would simply translate it to the right unit, and the logo can be made too.

However Ahmed's way does not give him the equation of curve 2.



To complete the above logo, curve 1 is transformed into curve 3, and curve 2 is transformed into curve 4

16. What single transformation is performed on curves 1 and 2 in order to end up with the logo?

Both curves 1 and 2 is reflected along the line  $y=6$ , which is the line of symmetry of curves 1, 2 and curves 3, 4.

17. What is the equation of curve 3?

$$y = (x-3)^2 + 2$$

$$y = -6(x-3)^2 - 12$$

$$y = -6[(x-3)^2 + 2]$$

**End of Assessment**